Indian Statistical Institute
Bangalore Centre
B.Math (Hons.) III Year 2012-2013
Second Semester
Sample Survey and Design of Experiments
Mid-semester Examination Date : 6.3.13

Answer as many questions as possible. The maximum you can score is 56.

All the notation have their usual meaning. State clearly the results you use.

1. Consider a simple random sample (SRS) of size n from a population of size N, collected **with replacement (WR)**.

   (a) Show that the sample mean is unbiased for population mean.

   (b) Obtain the expression for variance of the sample mean.

   (c) In order to estimate the proportion of smokers in a region with 25723 adult males, a simple random sample of size 200 was taken with replacement. 87 of them were found to be smokers.

   (i) Estimate the proportion of smokers in the population and find the standard error of your estimate.

   (ii) Do the results of the sample furnish conclusive evidence that the majority of the adult males are non-smokers.

   [State clearly your assumption with a short justification.]

   $[2 + 5 + ((1+2) + 6) = 16]$

2. (a) What is meant by "proportional allocation" in the context of stratified random sampling ?

   (b) A sampler used the method of stratified sampling applying SRSWR for each strata. Obtain the expression for the variance of the estimator of population mean.

   (c) Consider the following statement. " Stratified random sampling is better than simple random sampling only when stratification is done in the proper manner."

   (i) What are meant by the terms "better" and "proper" here ?

   (ii) Verify the truth of this statement for the "proportional allocation" scheme.

   $[\ 1 + 2 + (1 + 1 + 10) = 15]$

3. Consider a design with b blocks, each of size k, v treatments and incidence matrix N.

   (a)Write down an appropriate linear model.

(b) Denote the reduced normal equations for treatment effects by $C\hat{\tau} = Q$. Show that $l'\tau$ is estimable iff l is in the column space of C. Obtain the variance of the BLUE of $l'\tau$ assuming it to be estimable.

(c) Consider the following block design.

Bl	1	–	1	1	2	2
Bl	2	–	3	3	4	5
Bl	3	–	1	5	6	

Find whether the following treatment contrast are estimable.

(i)$\tau_1 - \tau_4$, (ii) $\tau_3 - \tau_6$.

[2+ (5+3) + 2 x 2 = 14]

4. It was desired to compare the milage performance of 4 petroleum products as fuel for cars. It was decided to use cars from 4 different companies and 4 different drivers. Money available was enough for 16 liters of fuel.

   (a) Explain how an experiment can be conducted with the help of a diagram. Write down an appropriate model.

   (b) Let τ_i denote the average milage performance of the ith petroleum product, $i = 1, 2, 3, 4$. Obtain an unbiased estimate of $\tau_i - \tau_j, i \neq j$.

   (c) Derive the sum of squares for testing the equality of treatment effects. [3 + 5 + 10 = 18]